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A cycle-based characterization of determinantal equivalence

Two bivariate functions $K, Q : \Lambda^2 \to \mathbb{F}$ are said to be determinantally equivalent if for any $n \in \mathbb{N}$ and $x_1, x_2, \dots, x_n \in \Lambda$, the determinants of matrices $(K(x_i, x_j))_{1 \le i,j \le n}$ and $(Q(x_i, x_j))_{1 \le i,j \le n}$ agree. We study to what extent such functions K and Q must be related by two canonical transformations corresponding to diagonal similarity and transposition.

In the finite setting, this is closely related to the classical problem from [R. Loewy, *Principal minors and diagonal similarity of matrices*, Linear Algebra and its Applications 78, 1986, pp. 23–64] concerning principal minors and diagonal similarity of matrices. While previous approaches rely heavily on linear-algebraic techniques, we present a fundamentally different and entirely combinatorial method.

The key idea is to analyze permutations appearing in determinant expansions as cycles in a graph. In contrast to previous work in the nonvanishing setting [H. Sapranidis Mantelos, *Determinantally equivalent nonzero functions*, Discrete Mathematics 349(6):115021, 2026], the presence of zeros requires substantially new combinatorial arguments, as the identities used in that setting are no longer available. From the resulting cycle structure, we construct two auxiliary functions whose cocycle properties encode the possible equivalence mechanisms between K and Q. The main step is then to prove a structural dichotomy for 3-cycles, showing that all such cycles necessarily fall into a single class. This ultimately yields a precise description of determinantally equivalent functions under a natural structural assumption.

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